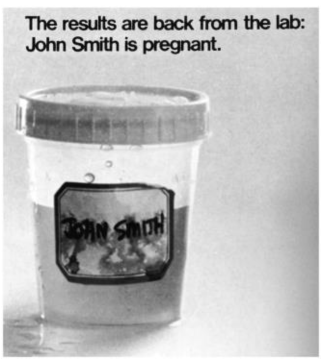
Chapter 3
Experimental Error

The derailment on October 22, 1895 of the Granville-Paris Express that overran the buffer stop. The engine careened across almost 100 feet (30 meters) of the station concourse, crashed through a two-foot thick wall, shot across a terrace and sailed out of the station, plummeting onto the Place de Rennes 30 feet (10 meters) below where it stood on its nose. While all of the passengers on board the train survived, one woman on the street below was killed by falling masonry.



Significant Figures

"The number of **significant figures** is the minimum number of digits needed to write a given value in scientific notation without loss of accuracy."

most significant figure - the left-hand most digit, the digit which is known most exactly

least significant figure - the right-hand most digit, the digit which is known most exactly

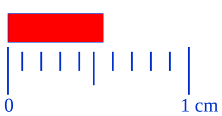
Counting Significant Figures

Rules for determining which digits are significant

1. All non-zero numbers are significant.
2. Zeros between non-zero numbers are significant.
3. Zeros to the right of the non-zero number **and** to the right of the decimal point are significant.
4. Zeros before non-zero numbers are **not** significant.

Significant Figures

When reading the scale of any apparatus, you should interpolate between the markings. It is usually possible to estimate to the nearest tenth of the distance between two marks.



0. 55 cm?

0. 56 cm?

2 signif. figs

0. 55 cm implies an error of at least 0.55 ± 0.01 cm

Significant Figures in Arithmetic

Exact numbers

conversion factors, significant figure rules do not apply

Significant Figures in Arithmetic

Addition and Subtraction

For addition and subtraction, the number of significant figures is determined by the piece of data with the fewest number of decimal places.

```
   4.371
+ 302.5
---------
 306.8
```

Significant Figures in Arithmetic

Multiplication and Division

For multiplication and division, the number of significant figures used in the answer is the number in the value with the fewest significant figures.

$$\frac{(2075)*(14)}{(144)} = 2.0 \times 10^2$$

Significant Figures in Arithmetic

Logarithms and Antilogarithms

logarithm of n:

n = 10^a ⟺ log **n** = **a**

n is the antilogarithm of **a**

log **339** = ***2.530***

2 ➡ character

.530 ➡ mantissa

Significant Figures in Arithmetic

Logarithms and Antilogarithms

The number of significant figures in the **mantissa** of the logarithm of the number should equal the number of significant figures in the number.

The **character** in the logarithm corresponds to the exponent of the number written in scientific notation.

Significant Figures in Arithmetic

Logarithms and Antilogarithms

The number of significant figures in the antilogarithm should equal the number of digits in the **mantissa**.

antilog (**-3.*42***) = $10^{-3.42}$ = **3.8 X 10^{-4}**

| | |

2 s.f. 2 s.f. 2 s.f.

Types of Error

Systematic Error (determinate error)

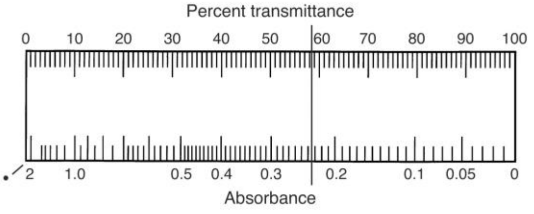
The key feature of systematic error is that, with care and cleverness, you can detect and correct it.

Types of Error

Types of Determinate Errors

- instrument error
- method errors
- personal errors

Analog Spectrometer



Types of Error

Effects of Determinate Errors

- constant errors

Detection of Determinate Instrument and Personal Errors

Types of Error

Detection of Determinate Method Errors

- analysis of standard samples (SRS)
- independent analysis
- blank determinations
- variation in sample size

Types of Error

Random Error (indeterminate error)

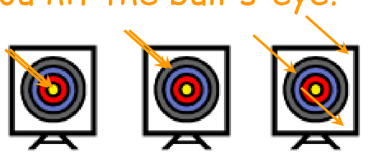
It is always present, cannot be corrected, and is the ultimate limitation on the determination of a quantity.

Types of Random Errors

- reading a scale on an instrument casued by the finite thickness of the lines on the scale
- electrical noise

Can you hit the bull's-eye?

Three shooters with three arrows each to shoot.



Both accurate and precise

Precise but not accurate

Neither accurate nor precise

Can you define accuracy and precision?

Precision and Accuracy

Precision

reproducibility

Accuracy (AKA bias)

closeness to accepted value

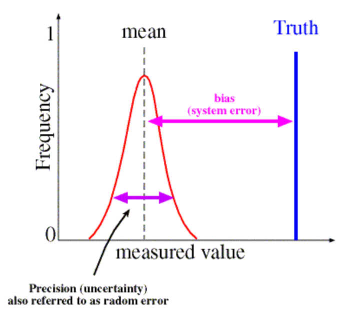
An ideal procedure provides both precision and accuracy.

Does that mean you can be precisely wrong? WHAT??

Absolute Uncertainty (Error)

- same units as measurement

absolute uncertainty = your value - true value



Relative Uncertainty (Error)

no units (ratio of numbers with same units)

$$\text{relative uncertainty} = \frac{\text{absolute uncertainty}}{\text{true value}}$$

$$\%\ \text{relative uncertainty} = \frac{\text{absolute uncertainty}}{\text{true value}} * 100$$

Propagation of Error

when possible uncertainty is expressed as a **standard deviation** or as a **confidence interval --- more on this later**

applies only to **random error**

Propagation of Error

Addition and Subtraction

uncertainty in addition and subtraction

$$e = \sqrt{e_1^2 + e_2^2}$$

$e = (e_1^2 + e_2^2 + e_3^2)^{1/2}$

percent relative uncertainty

$$\%\ e = \frac{\text{uncertainty}}{\text{mean}} * 100$$

ex. if absolute uncertainty = $\frac{0.04}{3.06} \times 100 = 1.3\ \%$

3.06 ± 0.04 (absolute)

3.06 ± 1 % (relative)

Propagation of Error

Multiplication and Division

$$e = x \cdot \sqrt{\left(\frac{e_1}{x_1}\right)^2 + \left(\frac{e_2}{x_2}\right)^2}$$

or

$\%e = ((\%e_1)^2 + (\%e_2)^2 + (\%e_3)^2)^{1/2}$

NOW FOR

The ***Real*** Rule of Significant Figures

The number of figures used to express a calculated result should be consistent with the uncertainty in that result.

Or – The answer should have the same number of decimal places as the ERROR..

Propagation of Uncertainty

Exponents and Logarithms

uncertainty for powers and roots

$y = x^a \Rightarrow \%e_y = a \%e_x$

Mixed Operations

Propagation of Uncertainty

Exponents and Logarithms

uncertainty for logarithm

y = log x $\rightarrow$ $e_y = \frac{1}{\ln 10}\frac{e^x}{x}$

uncertainty for antilogarithm

y = ln x $\rightarrow$ $e_y = \frac{e_x}{x}$

y = 10^x $\rightarrow$ $\frac{e_y}{y} = (\ln 10)\, e_x \approx 2.3026$

$\frac{e_y}{y}$=0.0691=$\frac{e_y}{6.17\times10^{-6}}$ $\rightarrow$ e_y=4.26 x 10^{-7}

Propagation of Uncertainty

Exponents and Logarithms

For pH= 5.21 ± 0.03 ,what is $[H^+]$ and its uncertainty?

$[H^+] = 10^{-pH} = 10^{-(5.21\pm0.03)}$

$\frac{e_y}{y} = 2.3026\, e_x$ = (2.3026)(0.03) = 0.0691

$\frac{e_y}{y}$=0.0691=$\frac{e_y}{6.17\times10^{-6}}$ $\rightarrow$ e_y=4.26 x 10^{-7}

The $[H^+]$ = 6.17 (±0.426) x 10^{-6} M $\rightarrow$ 6.2 (±0.4) x 10^{-6} M

An Uncertainty of 0.03 in pH gives a 7% uncertainty in $[H^+]$